Instructor: Dr Yifei Lou

Office: FO 2.408 E

Office hour: TBH + appt

Email:

I work at the intersection of computational math and data science

I have two thesis advisor

I took courses in CS/EE/Stat departments

. RA support in the summer is available if you impress me with your course work.

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Prerequisites

. Multi-variable calculus

. Linear algebra

. Programming (Matlab)

Learning objected

. Help students in understanding methods for solving scientific problems using computers.

. Understand the source, propagation, magnitude and rate of growth of errors introduced

. Preliminaries(ch1)

. Solving nonlinear equation (ch 3)

. Interpolation (ch4)

. integration (ch5)

Root finding problem: find xst. F(x) = 0

Min f(x)

Medthod to be studied.

. Bisection Medthod (3.1)

. Newton’s method (3.2)

. Secant method (3.3)

. Convergence analysis

. Comparison and discussion

**Interpolation (continue)**

. Polynomial interpolation

. (xi,yi) I = 1,…p

P(x)= anxn + an-a xn-1 + a01

P(xi) = yi

. Error analysis

. Data fitting -> Machine learning.

Overfitting, (high variance)

Underfitting (high bias)

Good balance (low bias, low variance)

. Riemann sum

. Methods to be studied

* Trapezoid method (5.1)
* Simpson’s rule (5.3)
* Gaussian Quadrature formula (5.4)

. Error analysis

Solving Ax = b

. Gaussian elimination (2.1)

. Pivoting (2.2)

. Tridiagonal and banded system (2.3)

. Matrix factorization (8.1)

. Midterm Oct 19 in class

. Any question.

**Chapter 1:**

Significant digits of precision

. Non-zero digits within given measurements are significant.

. Zeros to the right of the last non-zero digit are significant if within the measurement.

. Zeros to the left of the first nonzero digit are not significant.

. An exact number has an infinite number of significant digits (or figures).

EX: 0,0045 = sign = 2

1,0045 = sign = 5

0.1036 x+ 0,2122 y= 0.7381

0.2081 x + 0,4247 y = 0.9327

Y = - 547

Alpha = 0.2081 / 0.1036 =

. Rounding and chopping

. Rounding reduces the number of significant digits in a number.

. The result of rounding is a shorter number having fewer nonzero digits.

. The round to -even

Nested multiplication

. To evaluate the polynomial

P(x) = a0 + a1x + a2x2 + …

Pseudo -code:

Today ‘s Agenda

Taylor Series

.

Text

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**Use Taylor for computation**

A picture containing text

Description automatically generated

**Alternative:**

Text, letter

Description automatically generated

**Take Home message**

Fast convergence of a Taylor series can be expected near the point of expansion.

• Taylor series for 𝑓(𝑥) at a point c (not recommend)

Text, letter, whiteboard

Description automatically generated

Maclaurin series if 𝑐 = 0.

• How to compute? Horner’s algorithm.

**Deflation**

Graphical user interface, text, application

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Diagram

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Graphical user interface, text, application

Description automatically generated

Synthetic division

**Taylor Theorem**

Graphical user interface, text, application, email

Description automatically generated

**Mean value Theorem**

A special case of Taylor Theorem

Chart, line chart

Description automatically generated

**Alternating series**

Text, letter

Description automatically generated

• It only applies to alternating series.

• It gives an upper bound for the error.

• Back to ln 2 for an example.

A picture containing graphical user interface

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Graphical user interface

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We will discuss – Floating-point number system – Roundoff errors – Loss of significance

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